International Conference on Algebra and Discrete Mathematics
January 08-10, 2018

Title of the Paper

First Author[1], Second Author[2] and Third Author[3]

[1] First Author Affiliations with mailing address

[2] Second Affiliations with mailing address

[3] Third Author Affiliations with mailing address

Abstract

The abstract should summarize the context, content and conclusions of the paper in less than 200 words. It should not contain any references or displayed equations.

Keywords: Write maximum five keywords relevant to the topic.
Subject Classification: Write Mathematics Subject Classification numbers.

1 Introduction

The noncommutative tori is known to be the most accessible examples of noncommutative geometry developed by A. Connes [4]. A noncommutative torus is a universal C^*-algebra generated by two unitary operators subject only to a suitable commutation relation. It arises naturally in a number of different situations. Among others, it can be obtained as a deformation quantization of the algebra of continuous (smooth) functions on an ordinary torus. This approach has been used widely in gauge theories on noncommutative tori.

2 Some preliminaries on noncommutative complex tori

In this section, we review some basic facts for noncommutative complex tori and bundles on them.

2.1 Noncommutative complex two-tori

A noncommutative two-torus T_θ^2 is defined by two unitaries U_1, U_2 obeying the relation

$$U_1U_2 = \exp(2\pi i\theta)U_2U_1, \tag{1}$$

where $\theta \in \mathbb{R}/\mathbb{Z}$. The commutation relation (1) defines the presentation of the involutive algebra

$$A_\theta = \left\{ \sum_{n_1,n_2\in\mathbb{Z}^2} a_{n_1,n_2}U_1^{n_1}U_2^{n_2} \mid a_{n_1,n_2} \in \mathcal{S}(\mathbb{Z}^2) \right\},$$

where $\mathcal{S}(\mathbb{Z}^2)$ is the Schwartz space of sequences with rapid decay at infinity. The action of T^2 by translation on $C^\infty(T^2)$ gives an action of T^2 on A_θ. The infinitesimal form of the action defines a Lie algebra homomorphism $\delta : L \to \mathrm{Der}(A_\theta)$, where $L = \mathbb{R}^2$ is an abelian Lie algebra and $\mathrm{Der}(A_\theta)$ is the Lie algebra of derivations of A_θ. Generators δ_1, δ_2 of $\mathrm{Der}(A_\theta)$ act in the following way:

$$\delta_k \left(\sum_{(n_1,n_2)\in\mathbb{Z}^2} a_{n_1,n_2}U^{n_1}U^{n_2} \right) = 2\pi i \sum_{(n_1,n_2)\in\mathbb{Z}^2} n_k a_{n_1,n_2}U^{n_1}U^{n_2}. \tag{2}$$

A complex structure on T_θ^2 is defined in terms of a complex structure on the Lie algebra $L = \mathbb{R}^2$ which acts on A_θ. Let us fix a $\tau \in \mathbb{C}$ such that $\mathrm{Im}(\tau) \neq 0$ and then τ defines a one-dimensional subalgebra of $\mathrm{Der}(A_\theta)$ spanned by the derivation δ_τ given by

$$\delta_\tau \left(\sum_{(n_1,n_2)\in\mathbb{Z}^2} a_{n_1,n_2}U^{n_1}U^{n_2} \right) = 2\pi i \sum_{(n_1,n_2)\in\mathbb{Z}^2} (n_1\tau + n_2) a_{n_1,n_2}U^{n_1}U^{n_2}. \tag{3}$$

The noncommutative torus equipped with such a complex structure is denoted by $T_{\theta,\tau}^2$ and will be called a noncommutative complex torus.

2.2 Holomorphic bundles on $T_{\theta,\tau}^2$

Since the algebra A_θ is considered as the algebra of smooth functions on T_θ^2, the vector bundles on T_θ^2 correspond to finitely generated projective right A_θ-modules. Such modules can be constructed by the Heisenberg projective representations and it is known that for each $g = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \mathrm{SL}_2(\mathbb{Z})$ such that $d + c\theta \neq 0$, a Heisenberg module $E_g(\theta) := E_{d,c}(\theta)$ over A_θ is given by the Schwarz space $\mathcal{S}(\mathbb{R} \times \mathbb{Z}/c\mathbb{Z})$ equipped with the right action of A_θ:

$$fU_1(s,k) = f(s - \frac{d + c\theta}{c}, k-1), \quad fU_2(s,k) = \exp(s - \frac{kd}{c}) f(s,k),$$

where $s \in \mathbb{R}$ and $k \in \mathbb{Z}/c\mathbb{Z}$. For $g \in \mathrm{SL}_2(\mathbb{Z})$, the modules $E_g(\theta)$ will be referred as *basic modules*.

Proposition 2.1. *For a stable bundle E on X_τ whose topological type is specified by a matrix $g = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \mathrm{SL}_2(\mathbb{Z})$, there is a basic A_θ-module $E_g(\theta)$ equipped with a connection whose curvature is $-2\pi i\mu$.*

Proof. Associated to the curvature condition on the stable bundle E on X_τ, we define a Heisenberg commutation relation by

$$F_\nabla = [\nabla_1, \nabla_2] = -2\pi i\mu. \tag{4}$$

□

Acknowledgment

This work was financially supported by XXXXXXXX.

References

[1] S. Akbari, H.R. Maimani and S. Yassemi, When a zero-divisor graph is planar or a complete r-partite graph, *J. Algebra*, 270 (2003) 169-180.

[2] M.F Atiyah and I.G Macdonald, *Introduction to commutative algebra*, Addison-Wesley Publishing company (1969).

[3] D. F. Anderson and P. S. Livingston, The zero-divisor graph of a commutative ring, *J. Algebra*, 217 (1999) 434-447.

[4] A. Connes, *Noncommutative Geometry*, Academic Press, New York (1994).

[5] N. Ganesan, Properties of rings with a finite number of zero-divisors, *Math. Ann.*, 157 (1964) 215-218.

[6] A. T. White, The genus of the Cartesian product of two graphs, *J. Combin. Theory Ser. B*, 11 (1971) 89-94.